

Review

R102 – CompArch, Numbers, Dry Runs

Starter – binary addition

- The method for binary addition is the same as in any other number base (including decimal), except we get *much* more carrying. For the addition of 3_{10} and 14_{10} :

$$\begin{array}{rcccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ \hline \hline \end{array}$$

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- Try these:

$\begin{array}{rcccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}$	$\begin{array}{rcccc} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}$	$\begin{array}{rcccc} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array}$	$\begin{array}{rcccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}$	$\begin{array}{rcccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}$	$\begin{array}{rcccc} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{array}$

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0 0 0 1	1 0 1 1	0 1 1 1	0 0 1 0	1 1 0 1	1 0 0 1
0 0 0 0	1 1 1 1	0 1 1 1	0 0 1 0	0 0 1 1	1 1 0 1
0 0 1 0	1 0 1 0	1 1 1 0	0 1 0 0	0 0 0 1	0 1 1 0

Binary Addition and Subtraction & Two's Complement

Adding numbers together in base 2 and representing negative numbers

Specification

4.5.4.2 Unsigned binary arithmetic

Content

Be able to add two unsigned binary integers

4.5.4.3 Signed binary using two's complement

Content

Additional information

Know that signed binary can be used to represent negative integers and that one possible coding scheme is two's complement.

This is the only representation of negative integers that will be examined. Students are expected to be able to convert between signed binary and decimal and vice versa.

Know how to:

- represent negative and positive integers in two's complement
- perform subtraction using two's complement
- calculate the range of a given number of bits, n .

Overflow

- In a computer, binary numbers typically have a fixed length (number of bits)
- Arithmetic operations can, therefore, produce values that require more bits than we have space for
- This is called **overflow**
 - Often, we just ignore the overflow

Discussion

How could we represent negative numbers in binary?

Two's complement

- This is the typical method for working with 'signed' numbers in binary
 - The sign of a number tells us whether it is positive or negative
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-128	64	32	16	8	4	2	1

- If the sign bit is **1**, the number is **negative**
- If the sign bit is **0**, the number is **positive**

Representing negative numbers in two's complement

$$-45_{10}$$

-128	64	32	16	8	4	2	1
------	----	----	----	---	---	---	---

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1	1	0	1	0	0	1	1

Converting back to decimal is just the same as unsigned binary, but remember that the first column is negative:

$$-128 + 64 + 16 + 2 + 1 = -45$$



Practice

Convert -14 into two's complement binary
using 8 bits



Practice

Convert -14 into two's complement binary
using 8 bits

1111 0010

Practice



Convert -127 into two's complement
binary using 8 bits



Practice

Convert -127 into two's complement
binary using 8 bits

1000 0001

Practice



Convert 1011 1000 from two's complement binary to decimal

Practice



Convert 1011 1000 from two's complement binary to decimal

-72

Practice



Convert 0011 1000 from two's complement binary to decimal

Practice



Convert 0011 1000 from two's complement binary to decimal

56



‘Flipping the sign’

- Consider the number -44 in binary – how would we turn it into 44?

-128	64	32	16	8	4	2	1
1	1	0	1	0	1	0	0

- Trick:** Starting at the right, find all bits up to and including the first 1. Keep these bits the same, and flip the remaining bits.
- This works in reverse too



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Binary subtraction

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 - From GCSE maths, we know that $50 - 29 = 50 + (-29)$
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$$50_{10} = 0011\ 0010$$

$$-29_{10} = 1110\ 0011$$

- We can use binary addition to add these together, ignoring the overflow
 - $0001\ 0101 = 21_{10}$

Example 1

$$9 - 29$$

$$9_{10} = 0000\ 1001$$

$$-29_{10} = 1110\ 0011$$

Example 1

$$9 - 29$$

$$9_{10} = 0000\ 1001$$

$$-29_{10} = 1110\ 0011$$

$$1110\ 1100 = -20_{10}$$

Example 2

$$100 + 50$$

$$100_{10} = 0110\ 0100$$

$$50_{10} = 0011\ 0010$$

Example 2

$$100 + 50$$

$$100_{10} = 0110\ 0100$$

$$50_{10} = 0011\ 0010$$

$$1001\ 0110 = -106 \text{ (not } 150\text{?)}$$

Example 2

$$100 + 50$$

$$100_{10} = 0110\ 0100$$

$$50_{10} = 0011\ 0010$$

$$1001\ 0110 = -106 \text{ (not } 150\text{?)}$$

- With one byte using two's complement, we can only represent the range

$$-128 \text{ to } 127$$

Notes & practice

- Binary addition is the same as in decimal but with more **carrying**
- Computers often use fixed-width arithmetic where overflow is ignored
- Two's complement example: -29
 - Write the column headers with a negative sign bit header on the left
 - Keep adding numbers till we're done
 - Or, start positive and use the 'flipping the sign' method
- Subtraction in binary is just addition with two's complement negative numbers
- With 8 bits, we can only represent integers in the range -128 to 127 in two's complement integers

Try these problems:

1. Convert -56 to binary
2. Convert 1001 1111 into decimal
3. Compute $7 - 100$ in binary, showing your working
4. Using two's complements and 2 bits after the binary point, compute $10 - 19.25$ in binary, showing your working
5. What are the largest and smallest integers that can be represented using two's complement with two bytes?

Answers

1. Convert -56 to binary
 - 1001 1000
2. Convert 1001 1111 into decimal
 - -97
3. Compute $7 - 100$ in binary, showing your working
 - 1010 0011
4. Using two's complements and 2 bits after the binary point, compute $10 - 19.25$ in binary, showing your working
 - 110110.11
5. What are the largest and smallest integers that can be represented using two's complement with two bytes?
 - -32,768 to 32,767

Random Numbers in C#



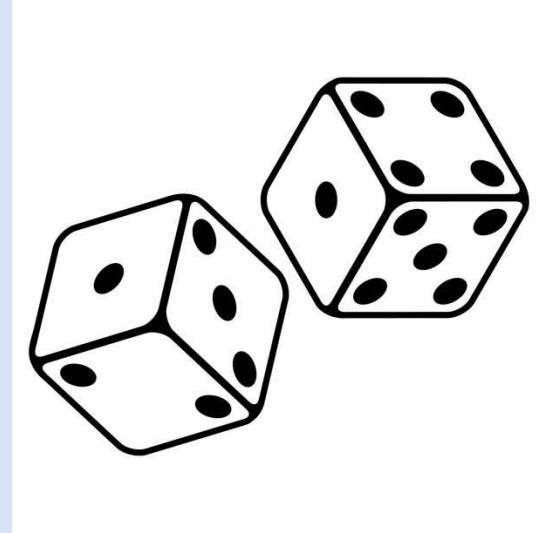
Discussion

In pairs, discuss answers to these questions and write your ideas on a whiteboard.

1. What is randomness?
2. What is a **random** number?
3. Can computers generate random numbers?
4. Why might we want to generate random numbers?

“True” randomness?

- Tossing a coin
- Rolling a die
- Roulette wheel
- Decay of an atom



Pseudorandom number generators

- A **pseudorandom number generator** (PRNG) is an algorithm that generates an apparently/observably random **sequence** of numbers

```
int getRandomNumber()  
{  
    return 4; // chosen by fair dice roll.  
              // guaranteed to be random.  
}
```

Seeding PRNGs

- A **seed** is a number used to initialise a pseudorandom number generator
- Given the same seed, two PRNGs will produce the same sequence of 'random' numbers

Seeding PRNGs



- A **seed** is a number used to initialise a pseudorandom number generator
- Given the same seed, two PRNGs will produce the same sequence of 'random' numbers
- Operating systems look for **entropy** (randomness through unpredictability)
 - Mouse movements
 - Keyboard timings
 - Variance in fan noise
- Cloudflare use an image feed of a rack of ~100 lava lamps to generate entropy

Random numbers in C#

Generating a random integer:

```
Random random = new Random(); // Seeded automatically  
Console.WriteLine(random.Next(1, 11));
```

This generates a random number from 1 to 10.

Ranges of random numbers

What ranges will these produce?

- `random.Next(1, 21);`
- `random.Next(1, 201);`
- `random.Next(2000, 2099);`
- `2 * random.Next(1, 51);`
- `Math.Pow(2, random.Next(1, 9));`

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- 1 to 20
 - 1 to 200

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- 1 to 20
 - 1 to 200
 - 2000 to 2098

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- 1 to 20
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 - Even numbers, 2 to 100

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 - `Math.Pow(2, random.Next(1, 9));`
- 1 to 20
 - 1 to 200
 - 2000 to 2098
 - Even numbers, 2 to 100
 - Powers of 2, 2 to 256

Reusing random number generators

- It is good practice to reuse a single random number generator
 - The version of C# we use uses the system clock to seed PRNGs, so creating multiple in close succession can lead to duplicate “random” numbers
 - It is always good practice to reuse them

```
Random random = new Random();  
int num1 = random.Next(1, 11);  
int num2 = random.Next(1, 11);
```

Practice

Example:

```
Random random = new Random();  
for (int i = 0; i < 20; i++)  
{  
    Console.WriteLine(random.Next(1, 11));  
}
```

You have **5 minutes** to see how far you can get.

Produce a list of 20 random numbers

between:

1. 1 to 30
2. 51 to 60
3. Even numbers, 0 to 10
4. Odd numbers, 1 to 11
5. Sums of powers of 3 and 5 up to $3^4 + 5^4$

Chance

- Sometimes, we are interested in probability
- We may have a random event that we want to occur 40% of the time in a game (e.g. a critical hit)

```
Random random = new Random();  
int value = random.NextDouble();
```

- `random.NextDouble()` generates a decimal x such that $0 \leq x < 1$

Worksheet

W105 - C# Random Numbers