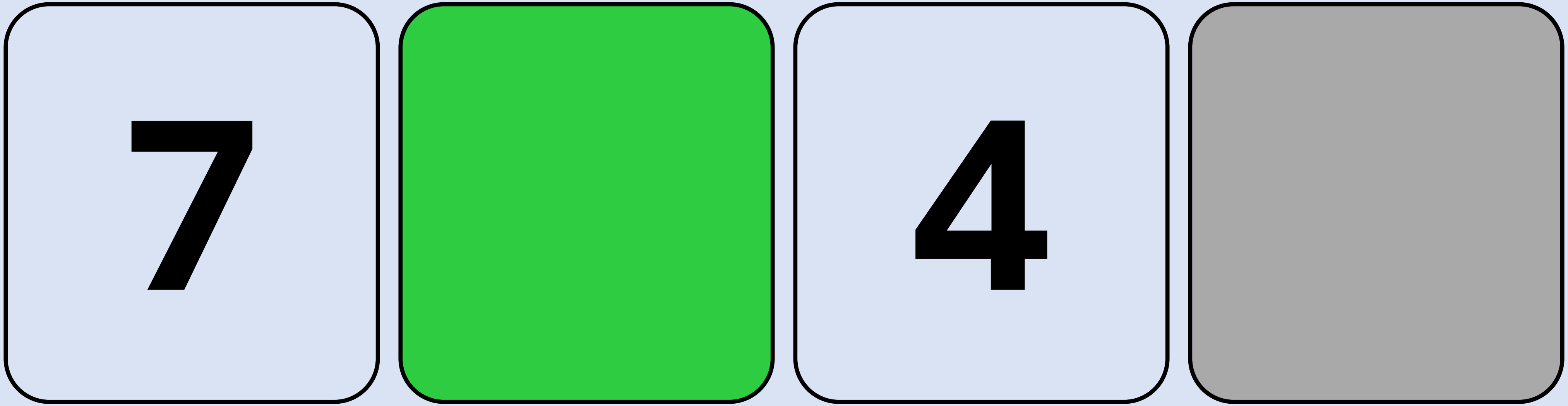
Bebras

- You will all be competing in the 2025 Bebras Computational Thinking Challenge in the second week after half term
- Practice using the Bebras 2020-2024 Elite challenges
  - Go to https://bebras.uk
  - Click on the **Challenges** tab
  - Click on **Elite** under **UK Bebras Challenge 2024**

Logic puzzle

You have some cards in front of you that definitely have one side numbered, and one side coloured.

Which cards do you have to turn over to test the rule, "*if a card has an odd number on one side, then the other side is grey*"?



Logic puzzle

Imagine you are a bouncer in Spoons. You see four people in front of you. What do you need to do?

1. Ask the age of the person drinking a coke
2. Ask the person, who is clearly in their 50s, what they are drinking
3. Ask the age of the person with a pint of lager
4. Ask what the person, who is clearly underage, is drinking

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These problems are logically equivalent.

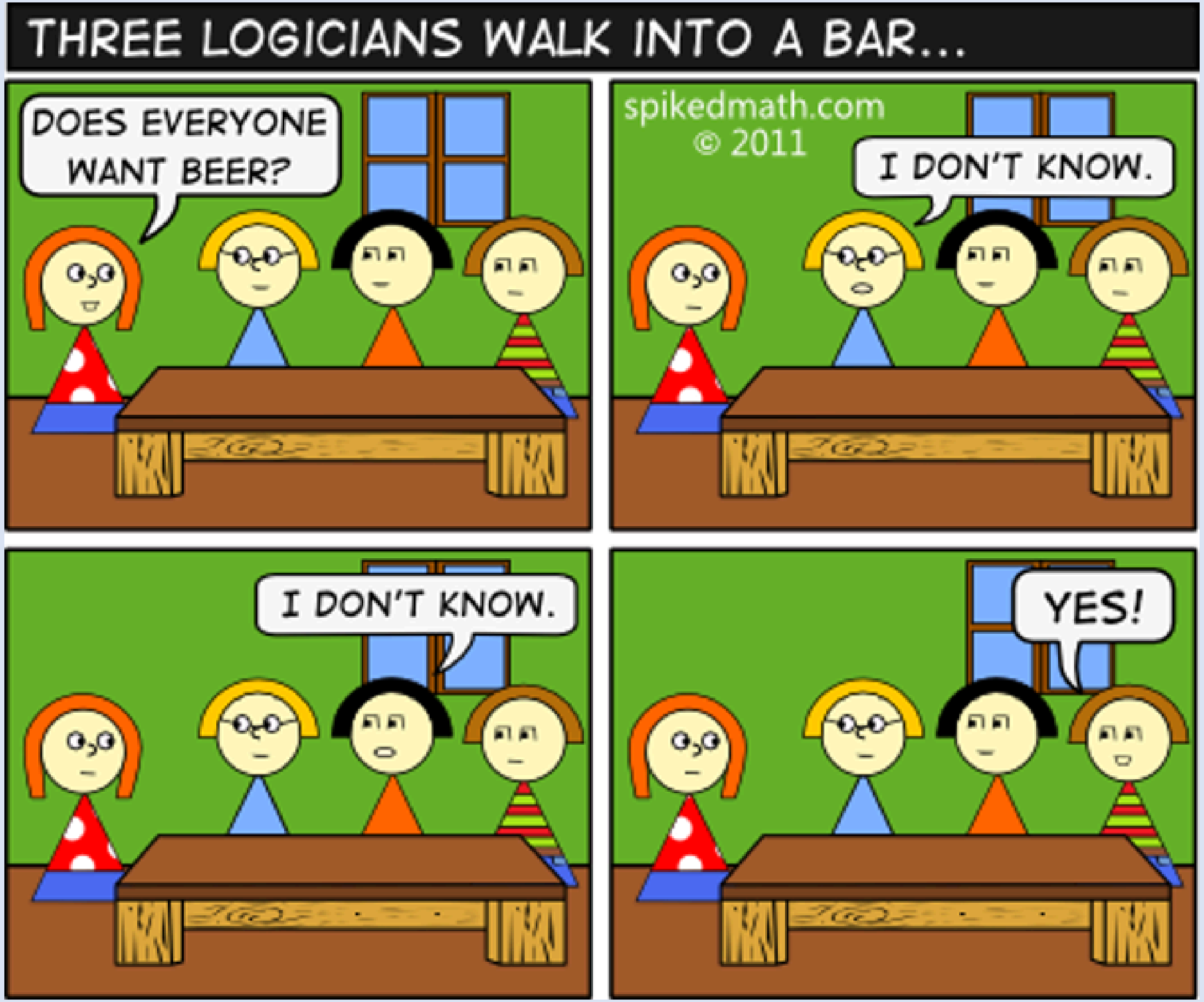
Logic puzzle

What can you conclude from the following information?

- All ducks in this village that are branded ‘B’ belong to Mrs Bond
- Ducks in this village never wear lace collars unless they are branded ‘B’
- Mrs Bond has no grey duck in this village

Lewis Carroll, “Symbolic Logic”, 1897

Logic puzzle


THREE LOGICIANS WALK INTO A BAR...
DOES EVERYONE WANT BEER?
spikedmath.com
© 2011
I DON'T KNOW.
I DON'T KNOW.
YES!

Logic Gates and Boolean Algebra

Specification

4.6.4.1 Logic gates

Content
Construct truth tables for the following logic gates: NOT; AND; OR; XOR; NAND; NOR.
Be familiar with drawing and interpreting logic gate circuit diagrams involving one or more of the above gates.
Complete a truth table for a given logic gate circuit.
Write a Boolean expression for a given logic gate circuit.
Draw an equivalent logic gate circuit for a given Boolean expression.

4.6.5.1 Using Boolean algebra

Content
Be familiar with the use of Boolean identities and De Morgan's laws to manipulate and simplify Boolean expressions.

Boolean expressions

- A **boolean expression** is a **condition** that can be evaluated to `TRUE` or `FALSE`
- Boolean expressions can be compounded by means of the operators `AND`, `OR` and `NOT`

Pre-work recall

Write out the truth table and draw the logic circuit symbols for:

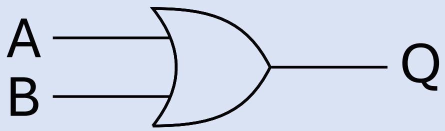
1. `OR` gate
2. `AND` gate
3. `NOT` gate

Logic gates

OR

$$Q = A + B$$

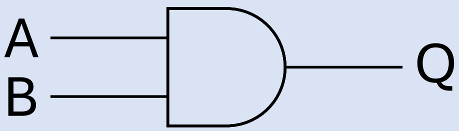
A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1



AND

$$Q = A \cdot B$$

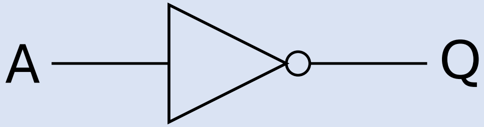
A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1



NOT

$$Q = \overline{A}$$

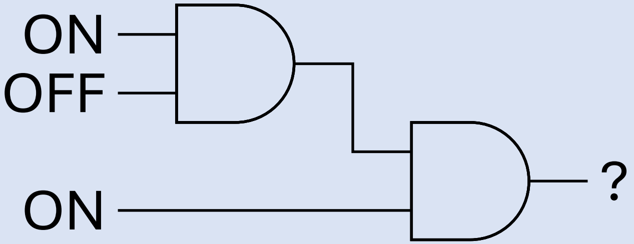
A	Q
0	1
1	0



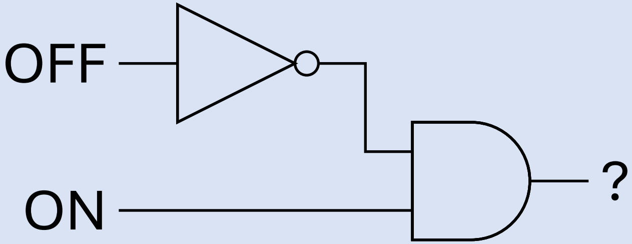
0 = OFF, 1 = ON

Practice

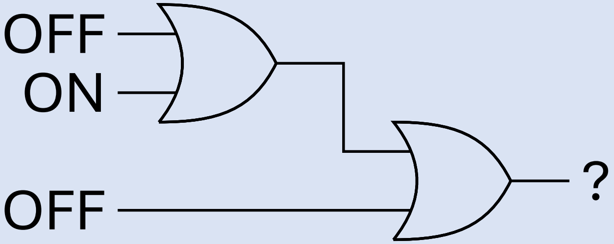
1.



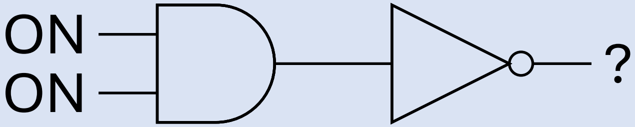
2.



3.

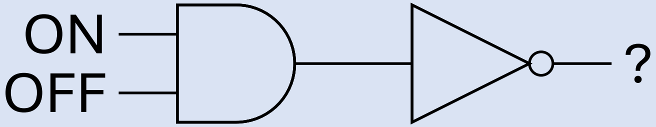


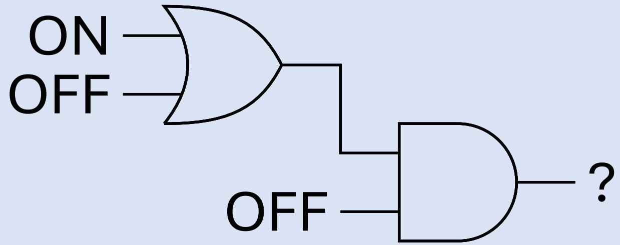
4.



5.

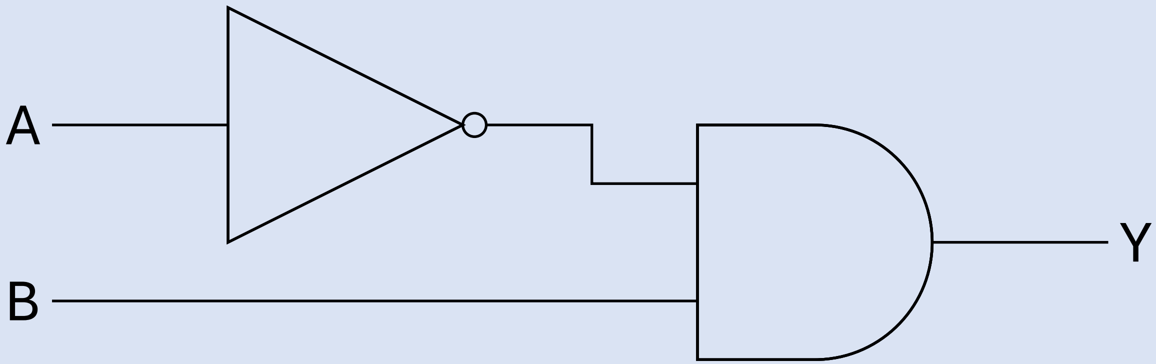
6.




ON
OFF
OFF
?

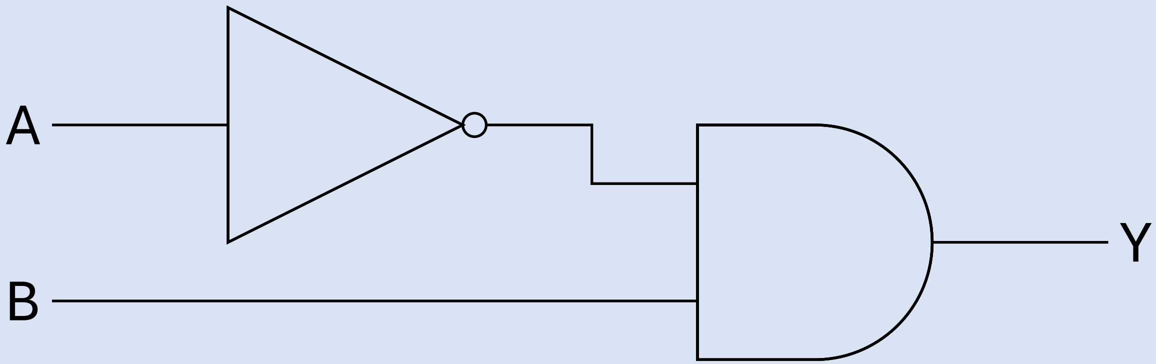
Practice

Write a Boolean expression for the following logic diagram.



Practice

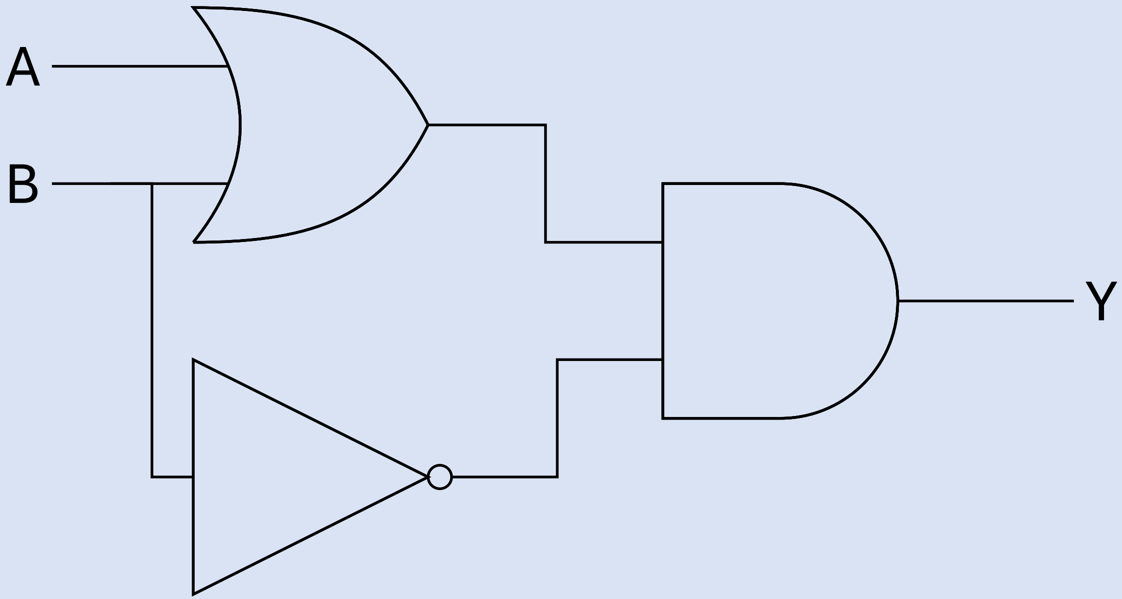
Write a Boolean expression for the following logic diagram.



$$Y = \overline{A} \cdot B$$

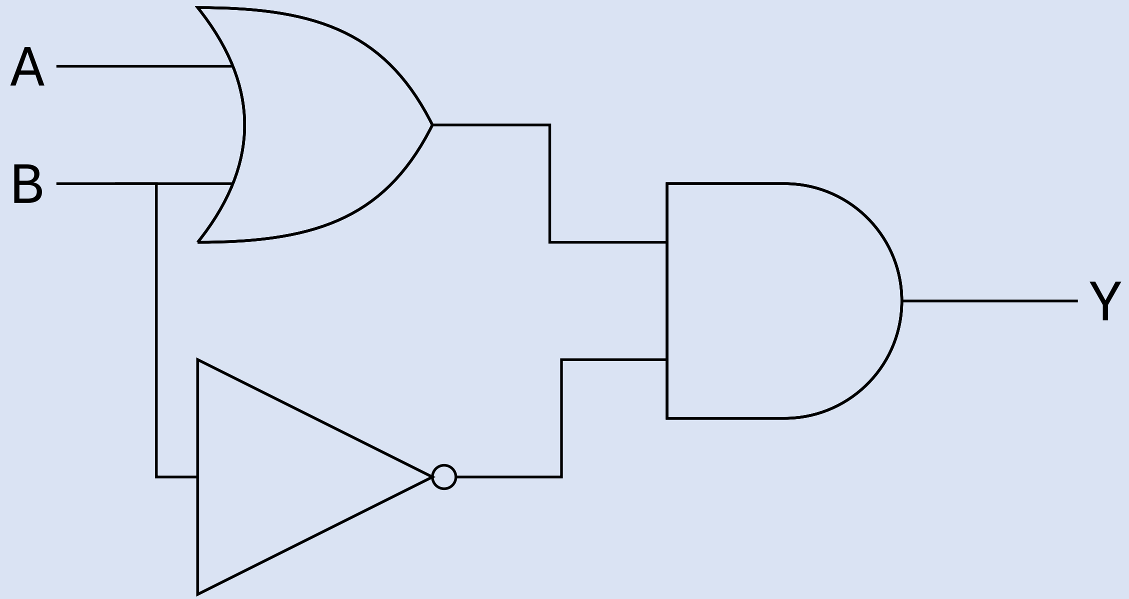
Practice

Write a Boolean expression for the following logic diagram.



Practice

Write a Boolean expression for the following logic diagram.



$$Y = (A + B) \cdot \overline{B}$$

Practice

Write a Boolean expression and draw a logic diagram for the following statement.

Tom will only go to Harry’s party if George and Victor are going, but not if Matt is going.

Pre-work recall

Write out the truth table and draw the logic circuit symbols for:

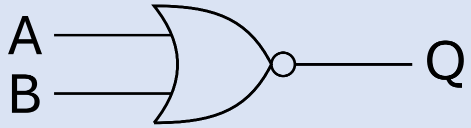
1. `NOR` gate
2. `NAND` gate
3. `XOR` gate

Logic gates

NOR

$$Q = \overline{A + B}$$

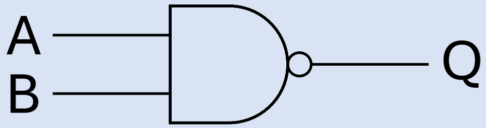
A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0



NAND

$$Q = \overline{A \cdot B}$$

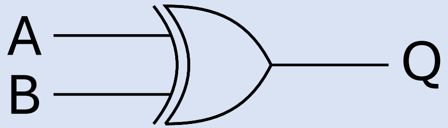
A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0



XOR

$$Q = A \oplus B$$

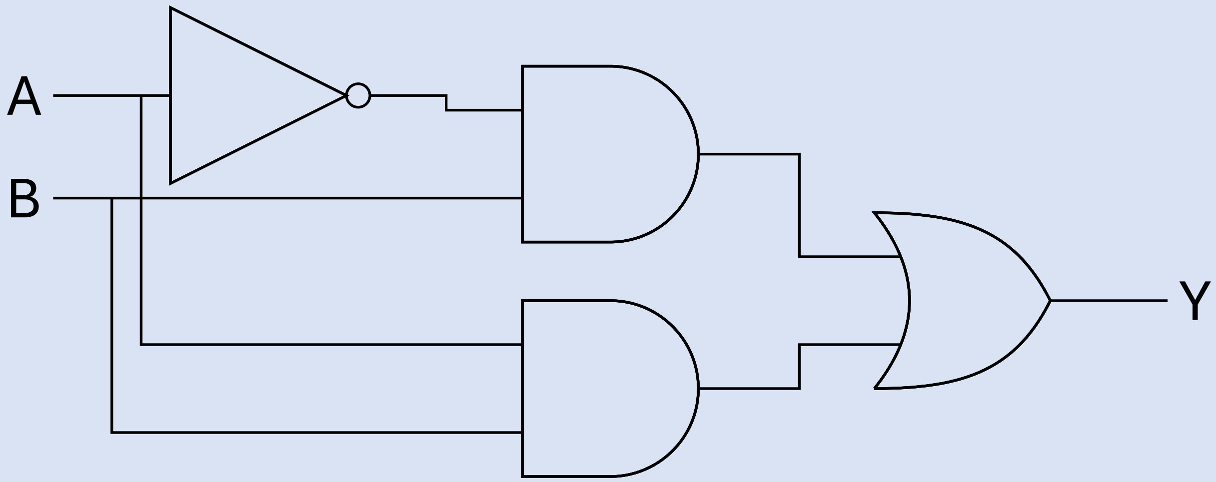
A	B	Q
0	0	0
0	1	1
1	0	1
1	1	0



$\boxed{0 = \text{OFF},\ 1 = \text{ON}}$

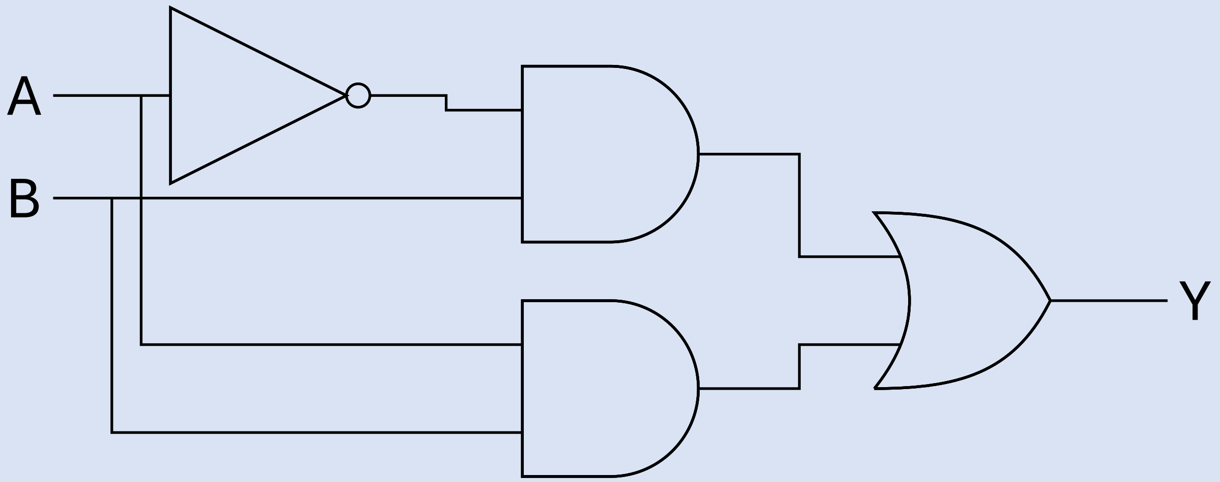
Practice

What is the Boolean equation for the following circuit?



Practice

What is the Boolean equation for the following circuit?



$$Y = \overline{A} \cdot B + A \cdot B$$

Motivating Boolean algebra

$$Y = \overline{A} \cdot B + A \cdot B$$

What is the truth table for this expression?

Motivating Boolean algebra

$$Y = \overline{A} \cdot B + A \cdot B$$

What is the truth table for this expression?

A	**B**	**Y**
0	0	
0	1	
1	0	
1	1	

Motivating Boolean algebra

$$Y = \overline{A} \cdot B + A \cdot B$$

What is the truth table for this expression?

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

What is this equivalent to?

Motivating Boolean algebra

$$Y = \overline{A} \cdot B + A \cdot B$$

What is the truth table for this expression?

A	B	Y
0	0	
0	1	
1	0	
1	1	

What is this equivalent to?

$$Y = B$$

Simplifying Boolean algebra

- Why?

Simplifying Boolean algebra

- Why?
  - Fewer logic gates → simpler circuits → cheaper designs

Simplifying Boolean algebra

- Why?
  - Fewer logic gates → simpler circuits → cheaper designs
- How?

Simplifying Boolean algebra

- Why?
  - Fewer logic gates → simpler circuits → cheaper designs
- How?
  - Using a truth table
  - Using Boolean algebra
    - The exam board prefer using Boolean algebra

Simplifying Boolean algebra

- Why?
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    - The exam board prefer using Boolean algebra

Maths skills required:

- Factorising brackets
- Expanding brackets

Laws of Boolean Algebra

Laws of Boolean Algebra

Commutative

$$A \cdot B = B \cdot A$$

$$A + B = B + A$$

Laws of Boolean Algebra

Commutative

$$A \cdot B = B \cdot A$$

$$A + B = B + A$$

Associative

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

$$A + (B + C) = (A + B) + c$$

Laws of Boolean Algebra

Commutative

$$A \cdot B = B \cdot A$$

$$A + B = B + A$$

Associative

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

$$A + (B + C) = (A + B) + c$$

Distributive

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$A + B \cdot C = (A + B) \cdot (A + C)$$

Boolean Algebra identities

- There is a set of Boolean identities which will help you to simplify expressions

$$0 + A =$$

Boolean Algebra identities

- There is a set of Boolean identities which will help you to simplify expressions

$$0 + A = A$$

Boolean Algebra identities

- There is a set of Boolean identities which will help you to simplify expressions

$$0 + A =$$

- Complete the worksheet on Google Classroom
  - Work in pairs, but complete both worksheets

Simplifying Boolean algebra expressions

1. Look at the expression
2. **Don’t panic!**
3. Use one of the **identities** or laws of **commutativity**, **associativity** or **distribution** to reduce one part of the expression
4. Repeat step 3 until there is nothing left to reduce

Examples

$$A + \overline{A} \cdot A$$

$$(A + B) \cdot (A + C)$$

$$B \cdot \left(A + \overline{B}\right)$$

$$A + A \cdot B$$

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$$A$$

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Examples

$$A + \overline{A} \cdot A$$

$$A$$

$$(A + B) \cdot (A + C)$$

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Examples

$$A + \overline{A} \cdot A$$

$$A$$

$$(A + B) \cdot (A + C)$$

$$A + B \cdot C$$

$$B \cdot \left(A + \overline{B}\right)$$

$$A \cdot B$$

$$A + A \cdot B$$

$$A$$

Practice

1. $A \cdot B + \overline{A} \cdot B$
2. $\overline{A} + B \cdot \overline{A}$
3. $\overline{A} \cdot (A + B)$
4. $B \cdot (A + A \cdot B)$
5. $(A + A) \cdot (A + B)$
6. $A + A \cdot A + \overline{A} \cdot B$

Practice

1. $A \cdot B + \overline{A} \cdot B$

   B

2. $\overline{A} + B \cdot \overline{A}$

   $\overline{A}$

3. $\overline{A} \cdot (A + B)$

   $\overline{A} \cdot B$

4. $B \cdot (A + A \cdot B)$

5. $(A + A) \cdot (A + B)$

6. $A + A \cdot A + \overline{A} \cdot B$

Practice

1. $A \cdot B + \overline{A} \cdot B$

   B

2. $\overline{A} + B \cdot \overline{A}$

   $\overline{A}$

3. $\overline{A} \cdot (A + B)$

   $\overline{A} \cdot B$

4. $B \cdot (A + A \cdot B)$

   $A \cdot B$

5. $(A + A) \cdot (A + B)$

   A

6. $A + A \cdot A + \overline{A} \cdot B$

   $A + B$