1. State the solution to

$$3 \ln x = 8$$

2. An account receiving compound interest can be modelled as

$$V = 1250(1 + 0.08)^t$$

where V is in dollars. State the initial value held in the account.

3. State the equation of the asymptote of the equation

$$y = 5^x + 43$$

4. Find the exact solution to

$$2 \log_{10} 7x + 1 = 7$$

5. Write

$$2x^3 - 3x^2 - 5x + 6$$

in the form

$$(x - 2)(ax^2 + bx + c)$$

1.

$$3\ln x = 8$$
$$x = e^{\frac{3}{8}}$$

2.

$$V = 1250(1+0.08)^t$$
$$V\big|_{t=0} = 1250$$

3.

$$y = 5^x + 43$$
$$y = 43$$

4.

$$2\log_{10} 7x + 1 = 7$$
$$2\log_{10} 7x = 6$$
$$\log_{10} 7x = 3$$
$$7x = 10^3 = 1000$$
$$x = \frac{1000}{7}$$

5.

$$2x^3 - 3x^2 - 5x + 6$$
$$= (x-2)(2x^2 + x - 3)$$

KA03 Question 4

It is given that $P(1, 5)$, $Q(8, 7)$ and $R(a, b)$ are collinear (they lie in a straight line). It is also given that $PR = 3PQ$. Find the possible coordinates of R.

Polynomials 2

Recap

Recap

Polynomial division

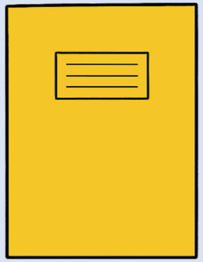
- We can use the grid method to divide polynomials by linear factors
- We can use this to write, for example, cubic functions as a product of three linear factors

Factor theorem

$$(x - a) \text{ is a factor of } f(x) \Leftrightarrow f(a) = 0$$

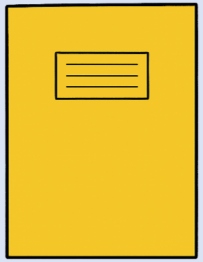
$$(bx - a) \text{ is a factor of } f(x) \Leftrightarrow f\left(\frac{a}{b}\right) = 0$$

Remainder theorem



If a polynomial $p(x)$ is divided by a linear factor $(x - \alpha)$, the remainder R is given by $R = p(\alpha)$.

Remainder theorem



If a polynomial $p(x)$ is divided by a linear factor $(x - \alpha)$, the remainder R is given by $R = p(\alpha)$.

Therefore, the **factor theorem** is a special case of the remainder theorem, whereby $R = p(\alpha) = 0$.

Practice

Polynomials – Factor Theorem

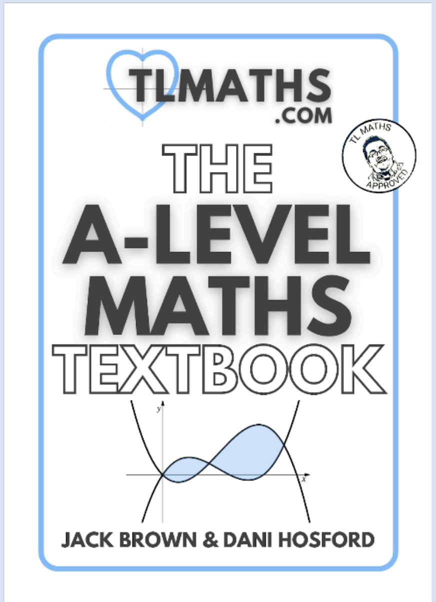
Page 30

Q13, Q14

Q15-23

Page 32

Q7



Extension Question

By using the factor theorem to find factors of $f(x)$, write

$$f(x) = 5x^5 + 2x^4 - 25x^3 - 10x^2 + 20x + 8$$

as a product of **five** linear factors.

Extension Question

By using the factor theorem to find factors of $f(x)$, write

$$f(x) = 5x^5 + 2x^4 - 25x^3 - 10x^2 + 20x + 8$$

as a product of **five** linear factors.

$$f(x) = (x-1)(x+1)(x-2)(x+2)(5x+2)$$

SA2 Revision

Q1 (a)

It is given that

$$f(x) = x^3 - x^2 + x - 6$$

Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

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$$f(x) = x^3 - x^2 + x - 6$$

Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

Q	Marking Instructions	Marks	Typical Solution
1(a)	Substitutes $x = 2$ into function	M1	$f(2) = 2^3 - 2^2 + 2 - 6$ $f(2) = 0$ which shows that $(x - 2)$ is a factor
	Completes reasoned argument to explain $f(2) = 0$ shows $(x - 2)$ is a factor	R1	
	Subtotal	2	

Q1 (b)

Find the quadratic factor of $f(x)$.

Q1 (b)

Find the quadratic factor of $f(x)$.

Q	Marking Instructions	Marks	Typical Solution
1(b)	Obtains correct factor	B1	$x^2 + x + 3$
	Subtotal	1	

Q1 (c)

Hence, show that there is only one real solution to $f(x) = 0$

Q1 (c)

Hence, show that there is only one real solution to $f(x) = 0$

Q	Marking Instructions	Marks	Typical Solution
1(c)	Calculates discriminant for their quadratic OE	M1	$x^2 + x + 3 = 0$ Discriminant $= 1^2 - 4 \times 1 \times 3$ $= -11 < 0$ So no real solutions to the quadratic Therefore $x = 2$ is the only solution
	States that there are no real solutions from the quadratic	A1	
	Deduces that there is only one solution coming from factor $(x - 2)$	B1	
	Subtotal	3	

Q1 (d)

Find the exact value of x that solves

$$e^{3x} - e^{2x} + e^{x} = 0$$

Q1 (d)

Find the exact value of x that solves

$$e^{3x} - e^{2x} + e^x = 0$$

Q	Marking Instructions	Marks	Typical Solution
1(d)	Expresses equation as a cubic in a single different variable or in terms of e^x $(e^x)^3 - (e^x)^2 + (e^x) - 6 = 0$	M1	$x^2 + x + 3$
	Obtains solution $e^x = 2$	A1	
	Obtains ln 2. ISW	A1	
	Subtotal	3	

Q2 (a)

A beaker containing a hot liquid at an initial temperature of 75°C cools so that the temperature, θ°C, of the liquid at time t minutes can be modelled by the equation

$$\theta = 5\left(4 + \lambda e^{-kt}\right)$$

where λ and k are constants.

After 2 minutes the temperature falls to 68°C.

Find the temperature of the liquid after 15 minutes. Give your answer to three significant figures.

Q2 (a)

Find the temperature of the liquid after 15 minutes. Give your answer to three significant figures.

Q	Marking Instructions	Marks	Typical Solution
2(a)	Uses model with $t = 0$ and $\theta = 75$ to form an equation	M1	
	Obtains correct λ	A1	
	Uses model with $t = 2$, $\theta = 68$ and their λ to form an equation	M1	
	Solves their equation correctly to find k	M1	

	Obtains correct k AWRT 0.07 **OE**	A1	$75 = 5\left(4 + \lambda e^{0}\right)$ $\lambda = 11$ $68 = 5\left(4 + 11e^{-2k}\right)$ $k = 0.068066$ $\theta = 5\left(4 + 11e^{-0.068066\times 15}\right)$ $= 39.8°\text{C}$
	Uses model with their λ and their k and $t = 15$	M1	
	Obtains correct temperature Condone missing units **AWRT** 39.8	A1	
	Subtotal	7	

Q2 (b) (i)

Find the room temperature of the laboratory, giving a reason for your answer.

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Find the room temperature of the laboratory, giving a reason for your answer.

Q	Marking Instructions	Marks	Typical Solution
2(b)(i)	States correct room temperature Condone missing units **CAO**	B1	$20°\mathrm{C}$ As t gets large the temperature predicted by the model will get close to room temperature.
	Explains that the temperature predicted by the model will approach room temperature as t increases **OE**	E1	
	Subtotal	2	

Q2 (b) (ii)

Find the time taken in minutes for the liquid to cool to 1 °C above the room temperature of the laboratory.

Q2 (b) (ii)

Find the time taken in minutes for the liquid to cool to 1 °C above the room temperature of the laboratory.

Q	Marking Instructions	Marks	Typical Solution
2(b) (ii)	Uses the model with their k and their room temperature $+1$ to form equation for t	M1	$5\left(4 + 11e^{-0.068066t}\right) = 21$ $t = 58.87$
	Obtains the correct value of t **AWRT 59** **ISW**	A1	
	Subtotal	2	

Q2 (c)

Explain why the model might need to be changed if the experiment was conducted in a different place.

Q2 (c)

Explain why the model might need to be changed if the experiment was conducted in a different place.

Q	Marking Instructions	Marks	Typical Solution
2(c)	Room temperature change/higher/ lower Cooling rate change/higher/lower or identifies a factor that may be different in a different place.	E1	The new room temperature might change
	Subtotal	1	

Plickers

Polynomials 2